

Radiative M1 transitions of heavy baryons in the bag model

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We study the M1 transitions of ground state heavy baryons within a framework of the modified bag model. Calculations of transition moments and corresponding M1 decay widths are performed. For the spin $\frac{1}{2}$ baryons containing three differently flavoured quarks the hyperfine mixing effects are taken into account. Results are compared with estimates obtained using various other approaches.

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I. INTRODUCTION

A study of electromagnetic properties of baryons plays an important role in the elementary particle physics. Electromagnetic observables serve as a source of information on the structure of hadrons. In our recent paper [1] we have used the modified bag model to calculate magnetic moments of $J = \frac{1}{2}$ and $J = \frac{3}{2}$ charmed and bottom baryons. For the baryons made of three differently flavoured quarks the colour-hyperfine mixing was taken into account. In the present paper we continue our exploration of heavy baryon electromagnetic structure through the calculation of their M1 transition moments. We seek to give some estimates for the radiative decay rates of heavy baryons as well. We expect it to be a useful step towards the comprehensive description of heavy baryon properties. Radiative decays of doubly heavy baryons have been studied (including hyperfine mixing effects) in nonrelativistic potential model [2] and in relativistic three-quark model [3]. We are going to compare our corresponding predictions with the results obtained in these papers.

The format of our paper is as follows. In Sec. II we give a brief overview of the model and present basic expressions necessary for our investigation. The results of our calculations for transition moments and decay widths are presented in Sec. III and Sec. IV, respectively. The last section is a short summary.

II. BAG MODEL AND TRANSITION MAGNETIC MOMENTS

The model we use to calculate the transition magnetic moments is exactly the same as has been used in our previous work [1]. For completeness we remind here the main features of this model (for details we refer to [4]), emphasizing its differences from the original MIT bag one [5].

The energy of the bag associated with a particular hadron is given by

$$E = \frac{4\pi}{3}BR^3 + \frac{Z_0}{R} + \sum_i \varepsilon_i + \Delta E, \quad (1)$$

where R denotes the bag radius, and the four terms on the right-hand side of this expression are: the bag volume energy, the Casimir energy, the sum of single-particle eigenenergies, and the quark-quark interaction energy due to one-gluon-exchange. The bag radius R_H of each hadron is obtained by minimizing (1) with respect to R .

We use the effective strong coupling constant and effective (running) quark mass. They are defined as

$$\alpha_c(R) = \frac{2\pi}{9 \ln(A + R_0/R)}, \quad (2)$$

$$\bar{m}_f(R) = \tilde{m}_f + \alpha_c(R) \cdot \delta_f. \quad (3)$$

The bag energy corrected for the center-of-mass motion (c.m.m.) is identified with the mass of hadron. It is related to the uncorrected one by

$$M^2 = E^2 - P^2, \quad (4)$$

where

$$P^2 = \gamma \sum_i p_i^2 \quad (5)$$

is the effective momentum square, and $p_i = \sqrt{\varepsilon_i - m_i}$ represent momenta of individual quarks.

The c.m.m. corrected magnetic moments are given by the relation

$$\mu = \frac{E}{M} \mu^0. \quad (6)$$

The model parameters are: the bag constant B , the Casimir energy parameter Z_0 , the parameter governing

the c.m.m. prescription γ , two parameters from the definition of the running coupling constant (A and R_0), and six parameters necessary to define the mass functions (Eq. 3) for the strange, charmed, and bottom quarks (\tilde{m}_f , δ_f). Light (u and d) quarks are assumed to be massless. All parameters are the same as in our previous work [1] ($B = 7.468 \times 10^{-4} \text{ GeV}^4$, $Z_0 = 0.22$, $\gamma = 2.153$, $A = 0.6514$, $R_0 = 4.528 \text{ GeV}^{-1}$, $\tilde{m}_s = 0.262 \text{ GeV}$, $\delta_s = 0.083 \text{ GeV}$, $\tilde{m}_c = 1.458 \text{ GeV}$, $\delta_c = 0.089 \text{ GeV}$, $\tilde{m}_b = 4.721 \text{ GeV}$, and $\delta_b = 0.079 \text{ GeV}$).

The magnetic moment of a quark confined in the bag of radius R_B is given by

$$\mu_i = q_i \bar{\mu}_i, \quad (7)$$

$$\bar{\mu}_i = \frac{4\varepsilon_i R_B + 2m_i R_B - 3}{2(\varepsilon_i R_B - 1)\varepsilon_i R_B + m_i R_B} \frac{R_B}{6}, \quad (8)$$

where q_i is the electric charge of the quark, and $\bar{\mu}_i$ denotes reduced (charge-independent) magnetic moment (see [5]).

The wave functions of ground state baryons can be constructed by coupling the spins of the two first quarks to an intermediate spin S and then adding the third one to obtain the total spin J :

$$|B\rangle = \left| (q_1 q_2)^{S=0} q_3, J = \frac{1}{2} \right\rangle, \quad (9a)$$

$$|B'\rangle = \left| (q_1 q_2)^{S=1} q_3, J = \frac{1}{2} \right\rangle, \quad (9b)$$

$$|B^*\rangle = \left| (q_1 q_2)^{S=1} q_3, J = \frac{3}{2} \right\rangle. \quad (9c)$$

The valence quark contribution to the baryon magnetic moments is [6]

$$\mu(B) = \mu_3, \quad (10a)$$

$$\mu(B') = \frac{1}{3}(2\mu_1 + 2\mu_2 - \mu_3), \quad (10b)$$

$$\mu(B^*) = \mu_1 + \mu_2 + \mu_3, \quad (10c)$$

where μ_i denote the magnetic moments of first, second, and third quarks, respectively. For the transition magnetic moments we have

$$\mu(B' \leftrightarrow B) = \frac{1}{\sqrt{3}}(\mu_2 - \mu_1), \quad (11a)$$

$$\mu(B^* \leftrightarrow B) = \sqrt{\frac{2}{3}}(\mu_1 - \mu_2), \quad (11b)$$

$$\mu(B^* \leftrightarrow B') = \frac{\sqrt{2}}{3}(\mu_1 + \mu_2 - 2\mu_3). \quad (11c)$$

The signs of transition moments depend on the adopted phase convention. Ours coincide with the one from Ref. [6].

Using Eqs. (10a)–(10c) and (11a)–(11c) the spin $\frac{3}{2} \leftrightarrow \frac{1}{2}$ transition moments can be expressed in terms of others:

$$\mu(B^* \leftrightarrow B') = \frac{\sqrt{2}}{3}[3\mu(B') - \mu(B^*)], \quad (12)$$

$$\mu(B^* \leftrightarrow B) = -\sqrt{2}\mu(B' \leftrightarrow B). \quad (13)$$

The baryons containing three quarks of different flavours need a special treatment. In this case the intermediate spin S is no longer a good quantum number. The colour-hyperfine interaction mixes the states with different intermediate spins, so that physical states are linear combinations of initial ones:

$$|B_{\text{phys}}\rangle = C_1 |B\rangle + C_2 |B'\rangle, \quad (14a)$$

$$|B'_{\text{phys}}\rangle = -C_2 |B\rangle + C_1 |B'\rangle. \quad (14b)$$

The physical (mixed) magnetic and transition moments are

$$\mu(B_{\text{phys}}) = C_1^2 \mu(B) + C_2^2 \mu(B') + 2C_1 C_2 \mu(B' \leftrightarrow B), \quad (15a)$$

$$\mu(B'_{\text{phys}}) = C_1^2 \mu(B') + C_2^2 \mu(B) - 2C_1 C_2 \mu(B' \leftrightarrow B), \quad (15b)$$

$$\begin{aligned} \mu(B'_{\text{phys}} \leftrightarrow B_{\text{phys}}) &= (C_1^2 - C_2^2) \mu(B' \leftrightarrow B) \\ &\quad + C_1 C_2 [\mu(B') - \mu(B)], \end{aligned} \quad (16)$$

$$\mu(B^*_{\text{phys}} \leftrightarrow B_{\text{phys}}) = C_1 \mu(B^* \leftrightarrow B) + C_2 \mu(B^* \leftrightarrow B'), \quad (17a)$$

$$\mu(B^*_{\text{phys}} \leftrightarrow B'_{\text{phys}}) = C_1 \mu(B^* \leftrightarrow B') - C_2 \mu(B^* \leftrightarrow B). \quad (17b)$$

The mixing of states is the reason that Eqs. (12) and (13), as they stand, are practically useless, because the states $|B\rangle$ and $|B'\rangle$ in general are not the physical states. These relations are valid only in the cases when the state mixing is absent or, at least, very small. Throughout, we work in the limit of exact isospin symmetry and thus neglect the small $\Sigma_Q - \Lambda_Q$ mixing. Other states unaffected by the hyperfine mixing are the remaining members of Σ_Q isomultiplet and the states corresponding to baryons Ω_Q , Ξ_{QQ} , Ω_{QQ} , where Q denotes heavy flavours (c and b). Such are also the ground states of triply heavy baryons Ω_{bcc}^+ and Ω_{bbc}^0 . For all of them Eq. (12) holds. Note that they all are of B' type (economy in primes leads to some mess-up in notations). As concerns Eq. (13), for the charmed baryon Σ_c^+ and bottom baryon Σ_b^0 (in the limit of exact isospin symmetry) we have

$$\mu(\Sigma_Q^* \leftrightarrow \Lambda_Q) = -\sqrt{2}\mu(\Sigma_Q \leftrightarrow \Lambda_Q), \quad (18)$$

where $\Sigma_Q = (\Sigma_c^+, \Sigma_b^0)$, and $\Lambda_Q = (\Lambda_c^+, \Lambda_b^0)$. In this case Λ_Q is of B type, and Σ_Q is of B' type.

Isospin symmetry also leads to some additional rela-

tions:

$$\mu(\Sigma_c^{*+} \leftrightarrow \Sigma_c^+) = \frac{1}{2} [\mu(\Sigma_c^{*0} \leftrightarrow \Sigma_c^0) + \mu(\Sigma_c^{*++} \leftrightarrow \Sigma_c^{++})], \quad (19a)$$

$$\mu(\Sigma_c^{*0} \leftrightarrow \Sigma_c^0) = -\frac{2\sqrt{2}}{3} \mu(\Sigma_c^{*+}), \quad (19b)$$

$$\mu(\Sigma_c^{*++} \leftrightarrow \Sigma_c^{++}) = -\frac{2\sqrt{2}}{3} \mu(\Sigma_c^{*0}), \quad (19c)$$

$$\mu(\Xi_{cc}^{*+} \leftrightarrow \Xi_{cc}^+) = \frac{\sqrt{2}}{3} \mu(\Xi_{cc}^{*++}), \quad (19d)$$

$$\mu(\Sigma_b^{*0} \leftrightarrow \Sigma_b^0) = \frac{1}{2} [\mu(\Sigma_b^{*-} \leftrightarrow \Sigma_b^-) + \mu(\Sigma_b^{*+} \leftrightarrow \Sigma_b^+)], \quad (20a)$$

$$\mu(\Sigma_b^{*-} \leftrightarrow \Sigma_b^-) = -\frac{2\sqrt{2}}{3} \mu(\Sigma_b^{*0}), \quad (20b)$$

$$\mu(\Sigma_b^{*+} \leftrightarrow \Sigma_b^+) = -\frac{2\sqrt{2}}{3} \mu(\Sigma_b^{*-}), \quad (20c)$$

$$\mu(\Xi_{bb}^{*-} \leftrightarrow \Xi_{bb}^-) = \frac{\sqrt{2}}{3} \mu(\Xi_{bb}^{*0}). \quad (20d)$$

Above we considered relations involving only the states with the same quark content. The naive quark model offers a somewhat richer collection of various relations including the states with different quark content, as, e. g.,

$$\mu(\Omega_{cc}^{*+} \leftrightarrow \Omega_{cc}^+) \approx -\mu(\Omega_c^{*0} \leftrightarrow \Omega_c^0), \quad (21)$$

$$\mu(\Omega_{bcc}^{*+} \leftrightarrow \Omega_{bcc}^+) \approx -\mu(\Omega_{bbc}^{*0} \leftrightarrow \Omega_{bbc}^0), \quad (22)$$

etc. However, in the bag model the magnetic moments of light (u, d), strange, and (to some extent) charmed quarks are sensitive to the environment in which they reside. Therefore the accuracy of such relations is very low. Equations (21) and (22) represent some kind of exception – the accuracy of the former (in our variant of the bag model) is $\approx 6\%$ and of the latter $\approx 3\%$. For all other relations of this type the accuracy is worse than 10%.

The relations presented above are a nice manifestation of underlying symmetry, however, in the absence of experimental data, at the time being we cannot check their validity in practice.

III. CALCULATION OF $\text{spin } \frac{3}{2} \leftrightarrow \frac{1}{2}$ TRANSITION MOMENTS

In this section we calculate the $\text{spin } \frac{3}{2} \leftrightarrow \frac{1}{2}$ transition magnetic moments of all ground state heavy baryons. Calculations are performed in the framework of modified bag model described in the previous section. Only the valence quark contribution is taken into account. The magnetic moments of quarks in the bag model depend on the bag radius of the hadron under consideration. Therefore in the case of transition moments a prescription which radius to use is necessary. One may try to

pick the smaller of the two with an intention to take into account the overlap of bags. However, for heavy baryons both radii are similar. We present the values of transition moments obtained using the radii of lighter baryons (B). The opposite choice may cause a shift of calculated values by less than 0.5%. Of course, such difference is irrelevant. Also we need the prescription how to use the c.m.m. correction (Eq. 6) because the ratio E/M for the baryons under transition may differ. We have checked that for charmed and bottom baryons the difference between E_{B^*}/M_{B^*} and E_B/M_B is sufficiently small, and we can choose any of the two. Our prescription was to use the ratio E_B/M_B corresponding to the lighter baryon again. The results of our calculations are listed in Tables I and II. We also compare our predictions with some other theoretical estimates. These are:

- Nonrelativistic quark model with screening and effective quark mass [7] (SCR).
- Chiral constituent quark model [8] (χ CQM).
- Light cone QCD sum rules [9] (LCSR). The definition of transition moment G_M used in that paper differs from ours. The relation between our results and theirs is $\mu = \sqrt{(3M_{B^*})/(2M_B)} (M_P/M_B) G_M$, where M_P is the mass of the proton. The factor M_P/M_B is used to convert natural magneton into nuclear magneton.
- Simple nonrelativistic quark model (NRQM). Since we have found few papers to compare our predictions with, we also give estimates obtained in non-relativistic quark model (with the state mixing accounted for the baryons containing three differently flavoured quarks). We treat these results as a kind of reference point. Input values for quark magnetic moments (in nuclear magnetons μ_N) and state mixing angles (in radians) were taken from Ref. [6]: $\mu_u = -2\mu_d$, $\mu_d = -0.93 \mu_N$, $\mu_s = -0.61 \mu_N$, $\mu_c = 0.39 \mu_N$, $\mu_b = -0.06 \mu_N$, $\theta_{usc} = \theta_{dsc} = 0.066$, $\theta_{usb} = \theta_{dsb} = 0.017$, $\theta_{ucb} = \theta_{dcb} = 0.13$, $\theta_{scb} = 0.12$.

We see from Table I that predictions given by the simple nonrelativistic model (NRQM) and the one with screening and effective quark mass (SCR) are in almost all cases similar. SCR results are, as a rule, slightly smaller (not more than 20%) than those obtained using NRQM. Only in two cases (for $\Xi_c^{*0} \leftrightarrow \Xi_c^0$ and $\Xi_c^{*+} \leftrightarrow \Xi_c^+$ transitions) the results differ significantly, and a large part of this difference comes from the hyperfine mixing effect. Predictions obtained using the chiral constituent quark model (χ CQM) are also similar to NRQM results (as a rule, slightly larger). As expected, predictions for $\Xi_c^{*0} \leftrightarrow \Xi_c^0$ and $\Xi_c^{*+} \leftrightarrow \Xi_c^+$ transitions differ significantly again. Results obtained using the light cone QCD sum rules (LCSR) for transitions $\Sigma_c^{*+} \leftrightarrow \Lambda_c^+$, $\Sigma_c^{*++} \leftrightarrow \Sigma_c^{++}$, $\Xi_c^{*0} \leftrightarrow \Xi_c^0$, and $\Xi_c^{*+} \leftrightarrow \Xi_c^+$ are compatible within the error bars with other predictions (χ CQM

TABLE I: Spin $\frac{3}{2} - \frac{1}{2}$ transition moments (in nuclear magnetons) of charmed baryons.

Transition	Our	NRQM	SCR [7]**	χ CQM [8]	LCSR [9]**
$\Sigma_c^{*0} \leftrightarrow \Sigma_c^0$	-1.030	-1.24	1.07	1.48	0.24 ± 0.05
$\Sigma_c^{*+} \leftrightarrow \Sigma_c^+$	-0.062	0.07	0.08	-0.003	0.57 ± 0.09
$\Sigma_c^{*+} \leftrightarrow \Lambda_c^+$	1.700	2.28	2.15	2.40	2.00 ± 0.53
$\Sigma_c^{*++} \leftrightarrow \Sigma_c^{++}$	0.905	1.39	1.23	-1.37	1.33 ± 0.38
$\Xi_c^{*0} \leftrightarrow \Xi_c^0$	-0.224	-0.33	0.18	-0.50	0.22 ± 0.07
$\Xi_c^{*0} \leftrightarrow \Xi_c^{'0}$	-0.915	-1.07	0.99	1.24	—
$\Xi_c^{*+} \leftrightarrow \Xi_c^+$	1.497	2.03	1.94	2.08	1.93 ± 0.72
$\Xi_c^{*+} \leftrightarrow \Xi_c^{' +}$	-0.089	0.09	0.17	-0.23	—
$\Omega_c^{*0} \leftrightarrow \Omega_c^0$	-0.839	-0.94	0.90	0.96	—
$\Xi_{cc}^{*+} \leftrightarrow \Xi_{cc}^+$	0.945	1.24	1.06	-1.41	—
$\Xi_{cc}^{*++} \leftrightarrow \Xi_{cc}^{++}$	-0.787	-1.39	1.35	1.33	—
$\Omega_{cc}^{*+} \leftrightarrow \Omega_{cc}^+$	0.789	0.94	0.88	-0.89	—

** Only absolute values $|\mu|$ are presented.

result for $\Xi_c^{*0} \leftrightarrow \Xi_c^0$ being an exception). For transitions $\Sigma_c^{*0} \leftrightarrow \Sigma_c^0$ and $\Sigma_c^{*+} \leftrightarrow \Sigma_c^+$ the LCSR predictions differ substantially from all other theoretical estimates. Such difference cannot be understood in the framework of the usual quark model and therefore looks strange.

Our predictions are, as a rule, smaller than other estimates but on average closer to results obtained in the nonrelativistic model with screening and effective quark mass (SCR) – of course, except the $\Xi_c^{*0} \leftrightarrow \Xi_c^0$, $\Xi_c^{*+} \leftrightarrow \Xi_c^+$ transitions. In these latter cases our results agree (at least qualitatively) with NRQM results, because these transitions are sensitive to the effect of state mixing. The reason why our results are in general smaller than others is also obvious. In many cases the contribution of the light quarks to the transition magnetic moments of heavy baryons is substantial. In the bag model, magnetic moments of the light quarks residing in heavy hadrons become smaller than in the light ones [1], and, as a consequence, one obtains relatively smaller transition moments.

In the bottom sector the transition moments obtained in the framework of the bag model are smaller than NRQM predictions again, but now they are in good agreement with LCSR results, while NRQM values are not. This is an indication that bag model predictions could be treated as serious improvement over NRQM results.

Magnetic moments of heavy quarks are not so sensitive to the environment they live in, and we expect the transition moments of baryons built up exclusively of heavy quarks to be similar in all models. Such are triply heavy baryons Ω_{bcc}^+ and Ω_{bbc}^0 . Indeed we see from Table II that the bag model predictions for these baryons differ from NRQM results approximately by only 5%.

Some aspects of our treatment still need some clarification. Maybe the most interesting question is the role of the colour-hyperfine mixing. The impact of the hyperfine state mixing on some electromagnetic properties of heavy baryons has been pointed out in [6]. Increasing interest in the heavy baryon spectroscopy made this problem more acute. The extensive study of the effect of colour-hyperfine mixing on the masses of heavy baryons [10, 11], semileptonic decays [12, 13], magnetic moments of heavy baryons [1], and electromagnetic decay rates [2, 3] was performed. The analysis is somewhat complicated by the dependence of wave functions on the arrangement of quarks in the spin coupling scheme $[[q_1 q_2]^S q_3]^J$ [6, 11]. There are three possible quark ordering schemes. The first is the scheme in which the quarks are ordered from lightest to heaviest, and the spins of the first two are coupled to the intermediate spin S . Let us call it a light diquark basis. The second scheme, in which the spins of the lightest and the heaviest quarks are coupled to the intermediate spin S , can be called a heavy-light diquark basis. The third, in which the two heaviest quarks are coupled to the intermediate spin, we will call a heavy diquark basis. Strictly speaking, these notations have little to do with real quark-diquark approximation. Just convenient names. In order to analyse the dependence on the choice of quark ordering we have performed calculations in all three ordering schemes. Expansion coefficients of the physical states in terms of initial wave functions with definite intermediate spins obtained in these calculations are presented in Table III.

From the table we see that physical states B and B' are predominantly light diquark states corresponding to intermediate spin $S = 0$ and $S = 1$, with a small admixture of other ($S = 1$ and $S = 0$) state. As expected [6], the largest state mixing is seen in the heavy-light

TABLE II: Spin $\frac{3}{2} - \frac{1}{2}$ transition moments (in nuclear magnetons) of bottom baryons.

Transition	Our	NRQM	LCSR [9]**
$\Sigma_b^{*-} \leftrightarrow \Sigma_b^-$	-0.504	-0.82	0.42 ± 0.14
$\Sigma_b^{*0} \leftrightarrow \Sigma_b^0$	0.345	0.49	0.20 ± 0.08
$\Sigma_b^{*0} \leftrightarrow \Lambda_b^0$	1.488	2.28	1.52 ± 0.58
$\Sigma_b^{*+} \leftrightarrow \Sigma_b^+$	1.193	1.81	0.83 ± 0.28
$\Xi_b^{*-} \leftrightarrow \Xi_b^-$	-0.139	-0.26	0.18 ± 0.06
$\Xi_b^{*-} \leftrightarrow \Xi_b^{'-}$	-0.415	-0.66	—
$\Xi_b^{*0} \leftrightarrow \Xi_b^0$	1.321	2.03	1.71 ± 0.60
$\Xi_b^{*0} \leftrightarrow \Xi_b^{'0}$	0.392	0.61	—
$\Omega_b^{*-} \leftrightarrow \Omega_b^-$	-0.339	-0.52	—
$\Xi_{bc}^{*0} \leftrightarrow \Xi_{bc}^0$	-0.747	-1.09	—
$\Xi_{bc}^{*0} \leftrightarrow \Xi_{bc}^{'0}$	0.070	-0.06	—
$\Xi_{bc}^{*+} \leftrightarrow \Xi_{bc}^+$	0.695	1.33	—
$\Xi_{bc}^{*+} \leftrightarrow \Xi_{bc}^{'+}$	0.672	0.95	—
$\Omega_{bc}^{*0} \leftrightarrow \Omega_{bc}^0$	-0.624	-0.82	—
$\Omega_{bc}^{*0} \leftrightarrow \Omega_{bc}^{'0}$	0.112	0.05	—
$\Omega_{bcc}^{*+} \leftrightarrow \Omega_{bcc}^+$	0.403	0.42	—
$\Xi_{bb}^{*-} \leftrightarrow \Xi_{bb}^-$	0.428	0.82	—
$\Xi_{bb}^{*0} \leftrightarrow \Xi_{bb}^0$	-1.039	-1.81	—
$\Omega_{bb}^{*-} \leftrightarrow \Omega_{bb}^-$	0.307	0.52	—
$\Omega_{bbc}^{*0} \leftrightarrow \Omega_{bbc}^0$	-0.395	-0.42	—

** Only absolute values $|\mu|$ are presented.

TABLE III: Expansion coefficients of the physical states in terms of wave functions with definite intermediate spins ($S = 0, 1$) in three different quark ordering schemes. q stands for the light quarks (u or d).

Particles	quark ordering	C_1	C_2
Ξ_c, Ξ_c'	$(qs)c$	0.997	0.073
"	$(cq)s$	-0.562	0.827
"	$(sc)q$	-0.435	-0.900
Ξ_b, Ξ_b'	$(qs)b$	0.999	0.018
"	$(bq)s$	-0.516	0.857
"	$(sb)q$	-0.484	-0.875
Ξ_{bc}, Ξ_{bc}'	$(qc)b$	0.992	0.128
"	$(bq)c$	-0.607	0.795
"	$(cb)q$	-0.385	-0.923
$\Omega_{bc}, \Omega_{bc}'$	$(sc)b$	0.994	0.112
"	$(bs)c$	-0.593	0.805
"	$(cb)s$	-0.400	-0.916

diquark case. The heavy diquark basis is somewhere between light diquark and heavy-light diquark. In the light diquark basis the mixing is very small. Nevertheless, as suggested in [6], and we have seen in [1], even such a small mixing can affect the magnetic moments as well as $B' - B$ transition moments appreciably. So we can anticipate the similar effect also in the case of spin $\frac{3}{2} \leftrightarrow \frac{1}{2}$ transition moments. In Tables IV and V we compare the predictions for these transition moments between physical states with unmixed moments calculated using wave functions corresponding to various quark ordering schemes. For the singly heavy baryons the results obtained using heavy-light diquark and heavy diquark schemes are of little interest (in this case all authors prefer to use the light diquark basis) and are omitted from the Table IV.

It is evident that the dependence of unmixed transition moments on the quark ordering is very strong (see Table V). For example, in the heavy diquark basis the doubly heavy baryon state Ξ_{bc}' usually is assumed to be the one with $S = 0$. The unmixed transition moments in this basis then are: $\mu(\Xi_{bc}^{*0} \leftrightarrow \Xi_{bc}^{'0}) = \mu(\Xi_{bc}^{*+} \leftrightarrow \Xi_{bc}^{'+}) = \mu(\Omega_{bc}^{*0} \leftrightarrow \Omega_{bc}^{'0}) = 0.352 \mu_N$. On the other hand, predictions for the physical states are: $\mu(\Xi_{bc}^{*0} \leftrightarrow \Xi_{bc}^0) = 0.070 \mu_N$, $\mu(\Xi_{bc}^{*+} \leftrightarrow \Xi_{bc}^+) = 0.672 \mu_N$, and $\mu(\Omega_{bc}^{*0} \leftrightarrow \Omega_{bc}^{'0}) = 0.112 \mu_N$.

We see that the best unmixed predictions are obtained in the light diquark basis. But even this best basis cannot be treated as sufficiently good. Only for the Ξ_b, Ξ_b' states the results are of rather high accuracy, in almost all other cases the account of the state mixing effect is important.

From Tables I and II we see that some moments are much smaller than others. Can we find the reason? This is the last point we want to discuss in this section. By the way, it is a fine example how various symmetry based considerations work.

Firstly, let us take a look at the $\Xi_c^{*0} \leftrightarrow \Xi_c^0$ and $\Xi_b^{*-} \leftrightarrow \Xi_b^-$ transitions forbidden by the U -spin symmetry. The U -spin is similar to isospin in that it is a symmetry in the exchange of d and s quarks, rather than u and d ones. This symmetry connects quarks (d and s) with the same charge and therefore is useful for the analysis of electromagnetic structure of hadrons. Since d and s quarks are the members of U -spin doublet in the case of exact U -spin symmetry one would have the relation for the reduced magnetic moments $\bar{\mu}_d = \bar{\mu}_s$ and analogous relation for magnetic moments ($\mu_d = \mu_s$). In turn, the isospin symmetry leads to a similar relation for the reduced magnetic moments of u and d quarks ($\bar{\mu}_u = \bar{\mu}_d$) but a different relation for quark magnetic moments (i. e., $\mu_u = -2\mu_d$). The explicit expression for the transition moment $\mu(\Xi_c^{*0} \leftrightarrow \Xi_c^0)$ in terms of reduced quark magnetic moments is

$$\mu(\Xi_c^{*0} \leftrightarrow \Xi_c^0) = \frac{\sqrt{2}}{3\sqrt{3}}(\bar{\mu}_s - \bar{\mu}_d), \quad (23)$$

and exactly the same holds for the transition $\Xi_b^{*-} \leftrightarrow \Xi_b^-$. The U -spin conservation would lead to $\mu(\Xi_c^{*0} \leftrightarrow \Xi_c^0) \rightarrow 0$

TABLE IV: Spin $\frac{3}{2} - \frac{1}{2}$ transition moments $\mu(B^* \leftrightarrow B)$ (in nuclear magnetons) of singly heavy baryons for the physical states (Ξ_Q, Ξ'_Q) and the states with definite intermediate spins in the light diquark basis.

State B	μ	State B	μ	State B	μ	State B	μ
Ξ_c^0	-0.224	Ξ_c^+	1.497	Ξ_b^-	-0.139	Ξ_b^0	1.321
$\Xi_c'^0$	-0.915	$\Xi_c'^+$	-0.089	$\Xi_b'^-$	-0.415	$\Xi_b'^0$	0.392
$ [ds]^0 c\rangle$	-0.158	$ [us]^0 c\rangle$	1.507	$ [ds]^0 b\rangle$	-0.132	$ [us]^0 b\rangle$	1.313
$ [ds]^1 c\rangle$	-0.931	$ [us]^1 c\rangle$	0.022	$ [ds]^1 b\rangle$	-0.418	$ [us]^1 b\rangle$	0.416

TABLE V: Spin $\frac{3}{2} - \frac{1}{2}$ transition moments $\mu(B^* \leftrightarrow B)$ (in nuclear magnetons) of doubly heavy baryons for the physical (mixed) states and the states in the light diquark, heavy-light diquark, and heavy diquark schemes.

State B	μ	State B	μ	State B	μ
Ξ_{bc}^0	-0.747	Ξ_{bc}^+	0.695	Ω_{bc}^0	-0.624
$\Xi_{bc}'^0$	0.070	$\Xi_{bc}'^+$	0.672	$\Omega_{bc}'^0$	0.112
$ [dc]^0 b\rangle$	-0.750	$ [uc]^0 b\rangle$	0.604	$ [sc]^0 b\rangle$	-0.632
$ [dc]^1 b\rangle$	-0.026	$ [uc]^1 b\rangle$	0.755	$ [sc]^1 b\rangle$	0.042
$ [bd]^0 c\rangle$	0.397	$ [bu]^0 c\rangle$	-0.956	$ [bs]^0 c\rangle$	0.280
$ [bd]^1 c\rangle$	-0.636	$ [bu]^1 c\rangle$	0.145	$ [bs]^1 c\rangle$	-0.570
$ [cb]^0 d\rangle$	0.352	$ [cb]^0 u\rangle$	0.352	$ [cb]^0 s\rangle$	0.352
$ [cb]^1 d\rangle$	0.662	$ [cb]^1 u\rangle$	-0.900	$ [cb]^1 s\rangle$	0.527

and $\mu(\Xi_b^{*-} \leftrightarrow \Xi_b^-) \rightarrow 0$. In real world the U -spin symmetry is broken ($\bar{\mu}_d > \bar{\mu}_s$), and these transition moments are to some extent suppressed but not strictly equal to zero. Moreover, they are enhanced by the hyperfine mixing effect (especially $\mu(\Xi_c^{*0} \leftrightarrow \Xi_c^0)$). Nevertheless, they still remain smaller than many others.

In the charm sector (see Table I) there are two really very small transition moments $\mu(\Sigma_c^{*+} \leftrightarrow \Sigma_c^+)$ and $\mu(\Xi_c^{*+} \leftrightarrow \Xi_c^+)$. The expression for the former assuming isospin symmetry (i.e., $\bar{\mu}_u = \bar{\mu}_d$) can be put in the form

$$\mu(\Sigma_c^{*+} \leftrightarrow \Sigma_c^+) = \frac{\sqrt{2}}{9}(\bar{\mu}_u - 4\bar{\mu}_c). \quad (24)$$

The small value of this transition moment means that an approximate relation $\bar{\mu}_u \approx 4\bar{\mu}_c$ holds. In a framework of the bag model it looks somewhat accidental. On the other hand, in the naive quark model the mass of the charmed quark is roughly four times larger than the effective mass of light quarks. In the nonrelativistic case $\mu_q \sim 1/m_q$, and therefore one can expect the value of $(\bar{\mu}_u - 4\bar{\mu}_c)$ to be rather small. It is also extra suppressed by the factor $\sqrt{2}/9$. Note that the usual magnetic moment of the doubly heavy baryon Ξ_{cc}^{++} given by the expression $\mu(\Xi_{cc}^{++}) = \frac{2}{9}(4\bar{\mu}_c - \bar{\mu}_u)$ (see Ref. [1]) is also much smaller than others.

The expression for $\mu(\Xi_c^{*+} \leftrightarrow \Xi_c'^+)$ is

$$\mu(\Xi_c^{*+} \leftrightarrow \Xi_c'^+) = \frac{\sqrt{2}}{9}(2\bar{\mu}_u - \bar{\mu}_s - 4\bar{\mu}_c). \quad (25)$$

In the limit of U -spin symmetry (the isospin symmetry is assumed also) $\bar{\mu}_u = \bar{\mu}_d = \bar{\mu}_s$, and Eq. (25) becomes equivalent to Eq. (24). Because the actual $\bar{\mu}_s$ is smaller than $\bar{\mu}_u$, we can expect $\mu(\Xi_c^{*+} \leftrightarrow \Xi_c'^+)$ to be larger than $\mu(\Sigma_c^{*+} \leftrightarrow \Sigma_c^+)$. This is true for unmixed moments, however the shift of $\mu(\Xi_c^{*+} \leftrightarrow \Xi_c'^+)$ due to the hyperfine mixing is negative, and this effect leads to an opposite relation $\mu(\Xi_c^{*+} \leftrightarrow \Xi_c'^+) < \mu(\Sigma_c^{*+} \leftrightarrow \Sigma_c^+)$. Since both these transition moments are negative, the absolute value of $\mu(\Xi_c^{*+} \leftrightarrow \Xi_c'^+)$ is larger.

In the bottom sector there also are two relatively small transition moments, i.e., $\mu(\Xi_{bc}^{*0} \leftrightarrow \Xi_{bc}'^0)$ and $\mu(\Omega_{bc}^{*0} \leftrightarrow \Omega_{bc}'^0)$. The expressions for these moments in the light diquark basis are

$$\mu(\Xi_{bc}^{*0} \leftrightarrow \Xi_{bc}'^0) = \frac{\sqrt{2}}{9}(2\bar{\mu}_c + 2\bar{\mu}_b - \bar{\mu}_u), \quad (26a)$$

$$\mu(\Omega_{bc}^{*0} \leftrightarrow \Omega_{bc}'^0) = \frac{\sqrt{2}}{9}(2\bar{\mu}_c + 2\bar{\mu}_b - \bar{\mu}_s). \quad (26b)$$

It is evident that transition moments $\mu(\Xi_{bc}^{*0} \leftrightarrow \Xi_{bc}'^0)$ and $\mu(\Sigma_c^{*+} \leftrightarrow \Sigma_c^+)$ have the same order of magnitude. We know that $\bar{\mu}_b < \bar{\mu}_c$. But also $\bar{\mu}_u(\Xi_{bc}) < \bar{\mu}_u(\Sigma_c)$, therefore the difference $|\mu(\Xi_{bc}^{*0} \leftrightarrow \Xi_{bc}'^0)| - |\mu(\Sigma_c^{*+} \leftrightarrow \Sigma_c^+)|$ cannot be large. $\mu(\Xi_{bc}^{*0} \leftrightarrow \Xi_{bc}'^0)$ and $\mu(\Omega_{bc}^{*0} \leftrightarrow \Omega_{bc}'^0)$ are also expected to be of the same order of magnitude because their difference $\frac{\sqrt{2}}{9}(\bar{\mu}_u - \bar{\mu}_s)$ vanishes in the U -spin symmetry limit. So we may anticipate the $\mu(\Xi_{bc}^{*0} \leftrightarrow \Xi_{bc}'^0)$ and $\mu(\Omega_{bc}^{*0} \leftrightarrow \Omega_{bc}'^0)$ to be small enough as their partners in the charm sector were. Both these transition moments undergo positive shifts due to the hyperfine mixing effect (see Table V), and $\mu(\Xi_{bc}^{*0} \leftrightarrow \Xi_{bc}'^0)$ even changes its sign. But they still remain smaller than other transition moments.

We have just seen how the light diquark basis facilitates the analysis of the electromagnetic properties of heavy baryons. Note that in the heavy diquark (as well as heavy-light diquark) basis the suppression of abovementioned transition moments is entirely a hyperfine mixing effect.

TABLE VI: Photon momenta (in MeV) calculated in the framework of the bag model (Our) and in a nonrelativistic potential model (PM) compared with an average value $\bar{\omega} = (\omega_{\text{Our}} + \omega_{\text{PM}})/2$ and with experimental data (Expt.).

Decay	Our	PM [10]	$\bar{\omega}$	Expt.
$\Sigma_c^* \rightarrow \Sigma_c$	88	63	76	63
$\Sigma_c^* \rightarrow \Lambda_c$	184	265	224	221
$\Sigma_c \rightarrow \Lambda_c$	99	180	139	162
$\Xi_c^* \rightarrow \Xi_c$	151	177	164	169
$\Xi_c^* \rightarrow \Xi'_c$	82	54	68	67
$\Xi'_c \rightarrow \Xi_c$	72	125	98	105
$\Omega_c^* \rightarrow \Omega_c$	75	57	66	72
$\Sigma_b^* \rightarrow \Sigma_b$	30	25	27	21
$\Sigma_b^* \rightarrow \Lambda_b$	155	241	198	209
$\Sigma_b \rightarrow \Lambda_b$	123	216	170	188
$\Xi_b^* \rightarrow \Xi_b$	124	171	148	152

IV. RADIATIVE DECAY WIDTHS

We wish to end our investigation with the predictions for radiative decay widths of ground state heavy baryons. We ignore E2 amplitudes which are expected to be much smaller than M1 transition moments (in the approximation we are using they are absent). The M1 partial width of the decay $B^* \rightarrow \gamma B$ has the form (see [14])

$$\Gamma = \frac{\alpha \omega^3}{M_P^2} \frac{2}{2J+1} \left(\frac{M_B}{M_{B^*}} \right) \mu^2(B^* \leftrightarrow B). \quad (27)$$

Here $\mu(B^* \leftrightarrow B)$ is the transition magnetic moment (in nuclear magnetons), $\alpha = \frac{1}{137}$, M_P is the proton mass. J and M_{B^*} are the spin and the mass of decaying baryon, M is the mass of the baryon in its final state, and

$$\omega = (M_{B^*}^2 - M_B^2)/(2M_B) \quad (28)$$

is the photon momentum in the c.m. system of decaying baryon.

In our calculations we have used transition moments from the preceding section for the spin $\frac{3}{2} \leftrightarrow \frac{1}{2}$ decays and transition moments obtained in our earlier paper [1] for $B' \rightarrow B$ decays. At present, in the absence of experimental data, we see no reliable way to estimate possible uncertainties of calculated transition moments and use them as they are. Another source of errors in the calculation of decay widths is the uncertainty in the value of photon momentum (28). The problem we are encountered with is that bag model predictions for baryon masses and corresponding mass differences are not of very high quality. Nevertheless, some regularities exist. One can check that the bag model almost always overestimates the baryon mass difference of $B^* - B'$ type. For example, such are $\Sigma_c^* - \Sigma_c$, $\Xi_c^* - \Xi'_c$, $\Omega_c^* - \Omega_c$, $\Sigma_b^* - \Sigma_b$ (see Table VI, values of photon momentum presented in this

TABLE VII: Radiative decay widths (in keV) of charmed baryons.

Decay	Our	Bag [17]	RQM [3, 18]	LCSR [9]
$\Sigma_c^{*0} \rightarrow \Sigma_c^0$	1.08	2.67	—	0.08 ± 0.03
$\Sigma_c^{*+} \rightarrow \Sigma_c^+$	0.004	1.52	0.14 ± 0.004	0.40 ± 0.16
$\Sigma_c^{*+} \rightarrow \Lambda_c^+$	126	176.7	151 ± 4	130 ± 45
$\Sigma_c^+ \rightarrow \Lambda_c^+$	46.1	22.91	60.7 ± 1.5	—
$\Sigma_c^{*++} \rightarrow \Sigma_c^{++}$	0.826	3.27	—	2.65 ± 1.20
$\Xi_c^{*0} \rightarrow \Xi_c^0$	0.908	—	0.68 ± 0.04	0.66 ± 0.32
$\Xi_c^{*0} \rightarrow \Xi'_c{}^0$	1.03	—	—	—
$\Xi'_c{}^0 \rightarrow \Xi_c^0$	0.0015	—	0.17 ± 0.02	—
$\Xi_c^{*+} \rightarrow \Xi_c^+$	44.3	74.01	54 ± 3	52 ± 25
$\Xi_c^{*+} \rightarrow \Xi'_c{}^+$	0.011	1.46	—	—
$\Xi'_c{}^+ \rightarrow \Xi_c^+$	10.2	—	12.7 ± 1.5	—
$\Omega_c^{*0} \rightarrow \Omega_c^0$	1.07	0.85	—	—
$\Xi_{cc}^{*+} \rightarrow \Xi_{cc}^+$	2.08	3.96	28.79 ± 2.51	—
$\Xi_{cc}^{*++} \rightarrow \Xi_{cc}^{++}$	1.43	4.35	23.46 ± 3.33	—
$\Omega_{cc}^{*+} \rightarrow \Omega_{cc}^+$	0.949	1.35	2.11 ± 0.11	—

table do not differ significantly from the corresponding mass differences). Furthermore, the remaining baryon mass differences of $B^* - B$ and $B' - B$ type are, as a rule, underestimated. An opposite tendency is seen in results obtained using the nonrelativistic potential model [10] (see Table VI again). To our knowledge this is the only paper that includes a full list of theoretical predictions for the masses of ground state heavy baryons we need. In order to minimize the uncertainties in the calculation of photon momenta we will use the experimental masses of baryons if available. The corresponding momenta are presented in Table VI (column Expt.) For all remaining transitions we use a somewhat arbitrary prescription which serves rather well in many cases where experimental masses are known. In the cases when experimental data are absent, our proposal is to use an average of our bag model and before-mentioned potential model result, $\bar{\omega} = (\omega_{\text{Our}} + \omega_{\text{PM}})/2$. To justify this choice, we compare in Table VI the bag model predictions for ω with potential model [10] predictions, average momenta $\bar{\omega}$, and experimental momenta (Expt.) calculated using experimental values of the baryon masses. The mass of Ξ_b^* is taken from [15], all others from Particle Data Tables [16].

We see that the averaged momentum in most cases is a significant improvement over the bag model predictions ($\Omega_c^* \rightarrow \Omega_c$ decay being an exception) and also over the potential model results (with the exception for the $\Sigma_c^* \rightarrow \Sigma_c$ and $\Sigma_b^* \rightarrow \Sigma_b$ decays). Therefore we expect that for other baryons (when data are absent) the averaged momentum also ought to be a reasonable choice.

TABLE VIII: Radiative decay widths (in keV) of singly heavy bottom baryons.

Transition	Our	LCSR [9]	LCSR-2**	HQET [19]**
$\Sigma_b^{*-} \rightarrow \Sigma_b^-$	0.010	0.11 ± 0.06	0.0076 ± 0.0041	0.020
$\Sigma_b^{*0} \rightarrow \Sigma_b^0$	0.005	0.028 ± 0.016	0.0017 ± 0.0009	0.0051
$\Sigma_b^{*0} \rightarrow \Lambda_b^0$	81.1	114 ± 45	84.5 ± 33.4	254
$\Sigma_b^0 \rightarrow \Lambda_b^0$	58.9	—	—	194
$\Sigma_b^{*+} \rightarrow \Sigma_b^+$	0.054	0.46 ± 0.22	0.030 ± 0.014	0.080
$\Xi_b^{*-} \rightarrow \Xi_b^-$	0.278	1.50 ± 0.75	0.464 ± 0.232	—
$\Xi_b^{*-} \rightarrow \Xi_b^{\prime -}$	0.005	—	—	—
$\Xi_b^{\prime -} \rightarrow \Xi_b^-$	0.118	—	—	—
$\Xi_b^{*0} \rightarrow \Xi_b^0$	24.7	135 ± 65	41.4 ± 20.6	—
$\Xi_b^{*0} \rightarrow \Xi_b^{\prime 0}$	0.004	—	—	—
$\Xi_b^{\prime 0} \rightarrow \Xi_b^0$	14.7	—	—	—
$\Omega_b^{*-} \rightarrow \Omega_b^-$	0.006	—	—	—

** Results obtained using current data for the masses of heavy baryons.

TABLE IX: Radiative decay widths (in keV) of doubly and triply heavy bottom baryons.

Transition	Our	RQM [3]	PM [2]
$\Xi_{bc}^{*0} \rightarrow \Xi_{bc}^0$	0.612	0.51 ± 0.06	1.03
$\Xi_{bc}^{*0} \rightarrow \Xi_{bc}^{\prime 0}$	0.0003	$(2 \pm 2) \times 10^{-6}$	0.0012
$\Xi_{bc}^{\prime 0} \rightarrow \Xi_{bc}^0$	0.125	0.31 ± 0.04	0.209
$\Xi_{bc}^{*+} \rightarrow \Xi_{bc}^+$	0.533	0.46 ± 0.10	0.739
$\Xi_{bc}^{*+} \rightarrow \Xi_{bc}^{\prime +}$	0.031	0.0015 ± 0.0007	0.061
$\Xi_{bc}^{\prime +} \rightarrow \Xi_{bc}^+$	0.037	0.14 ± 0.03	0.124
$\Omega_{bc}^{*0} \rightarrow \Omega_{bc}^0$	0.239	0.29 ± 0.03	0.502
$\Omega_{bc}^{*0} \rightarrow \Omega_{bc}^{\prime 0}$	0.0005	$(1 \pm 1) \times 10^{-6}$	0.0031
$\Omega_{bc}^{\prime 0} \rightarrow \Omega_{bc}^0$	0.053	0.21 ± 0.02	0.085
$\Omega_{bcc}^{*+} \rightarrow \Omega_{bcc}^+$	0.004	—	—
$\Xi_{bb}^{*-} \rightarrow \Xi_{bb}^-$	0.022	0.059 ± 0.014	—
$\Xi_{bb}^{*0} \rightarrow \Xi_{bb}^0$	0.126	0.31 ± 0.06	—
$\Omega_{bb}^{*-} \rightarrow \Omega_{bb}^-$	0.011	0.0226 ± 0.0045	—
$\Omega_{bbc}^{*0} \rightarrow \Omega_{bbc}^0$	0.005	—	—

To calculate the kinematical factor M_B/M_{B^*} we again use experimental masses when available. If experimental data are absent, we resort to the bag model results. As a consequence, our results for the radiative decay widths are not pure bag model predictions. But we think these improved results should be more accurate and therefore more useful. They are presented in Tables VII–IX. We also compare our predictions with results obtained using several other approaches. These are:

- Earlier MIT bag model predictions [17] (Bag).

- Nonrelativistic potential model [2] (PM).
- Relativistic three-quark model [3, 18] (RQM).
- Light cone QCD sum rules [9] (LCSR).
- LCSR estimates in the leading order of heavy quark effective theory [19] (HQET) calculated using current data for the masses of heavy baryons.

For charmed baryons (Table VII) the results obtained in all approaches form a varying pattern. In general most of them are more or less compatible. Nevertheless, there are exceptions. One can single out the LCSR prediction for $\Sigma_c^{*0} \rightarrow \Sigma_c^0$ decay width, which is an order of magnitude smaller than others, and a fussy mess-up in the $\Sigma_c^{*+} \rightarrow \Sigma_c^+$ decay rates predicted using various approaches. Another outstanding difference is predictions for the $\Xi_{cc}^{*+} \rightarrow \Xi_{cc}^+$ and $\Xi_{cc}^{*+} \rightarrow \Xi_{cc}^{\prime +}$ decay widths obtained in the relativistic three-quark model [3]. These are an order of magnitude larger than our predictions.

On the other hand, we predict very small decay widths for the M1 transitions $\Sigma_c^{*+} \rightarrow \Sigma_c^+$, $\Xi_c^{\prime 0} \rightarrow \Xi_c^0$, and $\Xi_c^{*+} \rightarrow \Xi_c^{\prime +}$. By the way, $\Sigma_c^{*+} \rightarrow \Sigma_c^+$ and $\Xi_c^{*+} \rightarrow \Xi_c^{\prime +}$ are the very same decays the small transition moments of which were discussed in detail in the preceding section.

In the case of singly heavy bottom baryons (Table VIII), when comparing our results with LCSR [9] predictions we are faced with an astonishing disagreement. Since transition moments in both approaches agree well (see Table II), we guess that in Ref. [9] the obsolete data for experimental baryon masses have been used. Therefore we recalculated these decay widths using the same transition moments (given in Ref. [9]) but with updated experimental values of baryon masses (in the calculation of photon momentum). The results are presented in the

column denoted as LCSR-2 of Table VIII. As expected, the corrected LCSR predictions are in satisfactory agreement with our results.

For doubly heavy B_{bc} and B_{bb} baryons (see Table IX) our predictions are compatible (at least qualitatively) with the estimates of radiative decay rates obtained using nonrelativistic potential model (PM) [2] and relativistic three-quark model (RQM) [3]. For example, all three models predict small decay widths for the transitions $\Xi_{bc}^{*0} \rightarrow \Xi_{bc}^{\prime 0}$ and $\Omega_{bc}^{*0} \rightarrow \Omega_{bc}^{\prime 0}$. Note that in all these approaches the state mixing due to colour-hyperfine interaction was taken into account. Our predictions for the decays $\Xi_{bc}^{*0} \rightarrow \Xi_{bc}^0$, $\Xi_{bc}^{*0} \rightarrow \Xi_{bc}^{\prime 0}$, $\Xi_{bc}^{*+} \rightarrow \Xi_{bc}^+$, $\Xi_{bc}^{*+} \rightarrow \Xi_{bc}^{\prime +}$, and $\Omega_{bc}^{*0} \rightarrow \Omega_{bc}^0$ are somewhere between decay rates obtained in PM and RQM. For other transitions we predict somewhat smaller decay widths. In Table IX we also give estimates for the decay rates of triply heavy baryons $\Omega_{bcc}^{*+} \rightarrow \Omega_{bcc}^+$ and $\Omega_{bcc}^{*0} \rightarrow \Omega_{bcc}^0$. They are relatively small, because small are the corresponding photon momenta ω .

V. SUMMARY

Using the modified bag model employed before in the study of magnetic moments of heavy baryons [1] we

have analysed the radiative decays of baryons containing one, two, and three heavy quarks. All heavy baryons are treated on the same footing. We have calculated the M1 transition moments for all ground state heavy baryons. These transition moments were used to obtain predictions for partial decay rates. To our knowledge for some transitions ($\Xi_b^{*-} \rightarrow \Xi_b^{\prime -}$, $\Xi_b^{\prime -} \rightarrow \Xi_b^-$, $\Xi_b^{*0} \rightarrow \Xi_b^0$, $\Xi_b^{\prime 0} \rightarrow \Xi_b^0$, $\Omega_b^{*-} \rightarrow \Omega_b^-$, $\Omega_b^{*+} \rightarrow \Omega_b^+$, and $\Omega_{bcc}^{*0} \rightarrow \Omega_{bcc}^0$) it is the first theoretical estimate. In the case of baryons containing three quarks of different flavours the state mixing due to colour-hyperfine interaction was taken into account. Because so far there are no experimental data to compare our predictions with, we have compared our results for magnetic transition moments and decay rates with those obtained in various other theoretical approaches. In many cases a good agreement was found. The existing differences were pointed out and in some cases the possible source of discrepancy was discussed.

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- [1] A. Bernotas and V. Šimonis, Magnetic moments of heavy baryons in the bag model reexamined, arXiv:1209.2900v1 (2012).
 - [2] C. Albertus, E. Hernández, and J. Nieves, Hyperfine mixing in electromagnetic decay of doubly heavy bc baryons, Phys. Lett. B **690**, 265 (2010).
 - [3] T. Branz, A. Faessler, T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij, and B. Oehl, Radiative decays of double heavy baryons in a relativistic constituent three-quark model including hyperfine mixing effects, Phys. Rev. D **81**, 114036 (2010).
 - [4] A. Bernotas and V. Šimonis, Towards the unified description of light and heavy hadrons in the bag model approach, Nucl. Phys. A **741**, 179 (2004).
 - [5] T. DeGrand, R. L. Jaffe, K. Johnson, and J. Kiskis, Masses and other parameters of the light hadrons, Phys. Rev. D **12**, 2060 (1975).
 - [6] J. Franklin, D. B. Lichtenberg, W. Namgung, and D. Carydas, Wave-function mixing of flavor-degenerate baryons, Phys. Rev. D **24**, 2910 (1981).
 - [7] R. Dhir and R. C. Verma, Magnetic moments of ($J^P = 3/2^+$) heavy baryons using effective mass and screened charge scheme, Eur. Phys. J. A **42**, 243 (2009).
 - [8] N. Sharma, H. Dahiya, P. K. Chatley, and M. Gupta, Spin $\frac{1}{2}^+$, spin $\frac{3}{2}^+$, and transition magnetic moments of low lying and charmed baryons, Phys. Rev. D **81**, 073001 (2010).
 - [9] T. M. Aliev, K. Azizi, and A. Ozpineci, Radiative decays of the heavy flavored baryons in light cone QCD sum rules, Phys. Rev. D **79**, 056005 (2009).
 - [10] W. Roberts and M. Pervin, Heavy baryons in a quark model, Int. J. Mod. Phys. A **23**, 2817 (2008).
 - [11] A. Bernotas and V. Šimonis, Mixing of heavy baryons in the bag model calculations, Lith. J. Phys. **48**, 127 (2008) [arXiv:0801.3570v1].
 - [12] W. Roberts and M. Pervin, Hyperfine mixing and the semileptonic decays of double-heavy baryons in a quark model, Int. J. Mod. Phys. A **24**, 2401 (2009).
 - [13] C. Albertus, E. Hernández, and J. Nieves, Exclusive $c \rightarrow s$, d semileptonic decays of ground-state spin-1/2 and spin-3/2 doubly heavy cb baryons, Phys. Rev. D **85**, 094035 (2012).
 - [14] G. Wagner, A. J. Buchmann, and A. Faessler, Electromagnetic properties of decuplet hyperons in a chiral quark model with exchange currents, J. Phys. G **26**, 267 (2000).
 - [15] S. Chatrchyan et al. (CMS Collaboration), Observation of a new Ξ_b baryon, Phys. Rev. Lett. **108**, 252002 (2012).
 - [16] J. Beringer et al. (Particle Data Group), Review of particle physics, Phys. Rev. D **86**, 010001 (2012).
 - [17] R. H. Hackman, N. G. Deshpande, D. A. Dicus, and V. L. Teplitz, M1 transitions in the MIT bag model, Phys. Rev. D **18**, 2537 (1978).
 - [18] M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij, and A. G. Rusetsky, Strong and radiative decays of heavy flavored baryons, Phys. Rev. D **60**, 094002 (1999).
 - [19] Shi-Lin Zhu and Yuan-Ben Dai, Radiative decays of heavy hadrons from light-cone QCD sum rules in the leading order of HQET, Phys. Rev. D **59**, 114015 (1999).